Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

https://sam.nitk.ac.in/ sam@nitk.edu.in

Probability Theory and Applications (MA208)
Problem Sheet - 6

Two and Higher Dimensional Random Variables

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1. Suppose that the following table represents the joint probability distribution of the discrete random variable (X, Y). Evaluate all the marginal and conditional distributions.

2. Suppose that the two-dimensional random variable (X, Y) has joint pdf

$$
\begin{aligned}
f(x, y) &= kx(x - y), \quad 0 < x < 2, \quad -x < y < x \\
&= 0, \quad \text{elsewhere.}
\end{aligned}
$$

Y \ X	1	2	3
1	$\frac{1}{12}$	$\frac{1}{6}$	0
2	0	$\frac{1}{9}$	$\frac{1}{5}$
3	$\frac{1}{18}$	$\frac{1}{4}$	$\frac{2}{15}$

(a) Evaluate the constant k. (b) Find the marginal pdf of X. (c) Find the marginal pdf of Y.

3. Suppose that the joint pdf of the two-dimensional random variable (X, Y) is given by

$$
\begin{aligned}
f(x, y) &= x^2 + \frac{xy}{3}, \quad 0 < x < 1, \quad 0 < y < 2, \\
&=, \quad \text{elsewhere.}
\end{aligned}
$$

Compute the following

(a) $P(X > \frac{1}{2})$ (b) $P(Y < X)$; (c) $P(Y < \frac{1}{2} | X < \frac{1}{2})$.

4. Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained and let Y be the number of queens obtained.

   (a) Obtain the joint probability distribution of (X, Y).
   (b) Obtain the marginal distribution of X and of Y.
   (c) Obtain the conditional distribution of X (given Y) and of Y (given X).

5. For what value of k is $f(x, y) = ke^{-(x+y)}$ a joint pdf of (X, Y) over the region $0 < x < 1, 0 < y < 1$?

6. Suppose that the continuous two-dimensional random variable (X, Y) is uniformly distributed over the square whose vertices are $(1, 0), (0, 1), (-1, 0)$, and $(0, -1)$. Find the marginal pdf's of X and of Y.

7. Suppose that the dimensions, X and Y, of a rectangular metal plate may be considered to be independent continuous - random variables with the following pdf's of X and of Y.

8. Suppose that the dimensions, X and Y, of a rectangular metal plate may be considered to be independent continuous random variables with the following pdf's.

$$\begin{aligned} x: \quad g(x) &= x-1, \quad 1 < x \leq 2, \\ &= -x+3, \quad 2 < x < 3, \\ &= 0, \quad \text{elsewhere.} \\ Y: h(y) &= \frac{1}{2}, \quad 2 < y < 4, \\ &= 0, \quad \text{elsewhere.} \end{aligned}$$

Find the pdf of the area of the plate, $A = XY$.

9. Let X represent the life length of an electronic device and suppose that X is a continuous random variable with pdf

$$\begin{aligned} f(x) &= \frac{1000}{x^2}, \quad x > 1000, \\ &= 0, \quad \text{elsewhere.} \end{aligned}$$

Let X_1 and X_2 be two independent determinations of the above random variable X. (That is, suppose that we are testing the life length of two such devices.) Find the pdf of the random variable $Z = X_1/X_2$.

10. Obtain the probability distribution of the random variables V and W introduced on p. 95.

11. Prove Theorem 6.1.

12. The magnetizing force H at a point P, X units from a wire carrying a current I, is given by $H = 2I/X$. (See Fig. 6.14.) Suppose that P is a variable point. That is, X is a continuous random variable uniformly distributed over $(3,5)$. Assume that the current I is also a continuous random variable, uniformly distributed over $(10,20)$. Suppose, in addition, that the random variables X and I are independent. Find the pdf of the random variable H.

13. The intensity of light at a given point is given by the relationship $I = C/D^2$, where C is the candlepower of the·source and D is the distance that the source is from the given point. Suppose that C is uniformly distributed over $(1,2)$, while D is a continuous random variable with pdf $f(d) = e^{-d}, d > 0$. Find the pdf of I, if C and D are independent. [Hint: First find the pdf of D^2 and then apply the results of this chapter.]

14. When a current I (amperes) flows through a resistance R (ohms), the power generated is given by $W = 1^2R$ (watts). Suppose that I and Rare independent random variables with the following pdf's.

$$\begin{aligned} I: \quad f(i) &= 6i(1-i), \quad 0 \leq i \leq 1, \\ &= 0, \quad \text{elsewhere.} \\ R: \quad g(r) &= 2r, \quad 0 < r < 1, \\ &= 0, \quad \text{elsewhere.} \end{aligned}$$

Determine the pdf of the random variable Wand sketch its graph.

15. Suppose that the joint pdf of (X, Y) is given by

$$f(x,y) = e^{-y}, \quad \text{for} x > 0, \quad y > x,$$
$$= 0, \quad \text{elsewhere.}$$

(a) Find the marginal pdf of X. (b) Find the marginal pdf of Y. (c) Evaluate $P(X > 2|Y < 4)$.

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